電動力學(一)題庫

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(Ch.1) Prove the following:

(a) There cannot be any static electric field in the interior (the hollow region) of a closed, hollow conductor if there is no charge in the interior.

(a) A closed, hollow conductor cannot shield its exterior from the electric fields due to charges placed in its interior.

(Ch. 1) Consider two concentric spherical metal shells of radii r_1 and $r_2 (r_2 > r_1)$. If the outer shell has a total charge Q_2 and the inner shell is grounded, what is the total charge Q_1 on the inner shell?

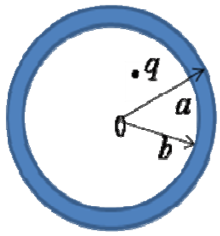
(Ch. 1) Consider an insulated, hollow conductor of outer radius a and inner radius b. The total charge on the conductor is zero. A point charge q is placed in the hollow region of the conductor.

(a) What is the total charge on the *inner* surface of the conductor? Why?

(b) What is the total charge on the *outer* surface of the conductor?

(c) What is the surface charge density σ on the outer surface of the conductor? Why?

(d) Use the proper theorem (or law) to calculate the electric field **E** in the region $r > a$.



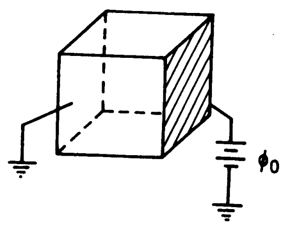
(Ch. 1) Calculate the work required to assemble a spherical charge cloud of radius a and uniform density ρ_0.

(Ch. 1) A simple capacitor is a device formed by two insulated conductors adjacent to each other. If equal and opposite charges are placed on the conductors, there will be a certain difference of potential between them. The ratio of the magnitude of the charge on one conductor to the magnitude of the potential difference is called the capacitance (in SI units it is measured in farads). The insulating medium between the conductors is a dielectric with electrical permittivity ε. Using Gauss's law, calculate the capacitance of two thin, flat, conducting plates of large area A, separated by a small distance d. Assume no fringe field (i.e. all filed lines are parallel to the plates.

(Ch. 1) Calculate the electric field energy density of the static electric field: $E = 6$ kV/cm.

(Ch. 1) Prove mathematically that the electrostatic potential (ϕ) in a charge-free region has no maximum or minimum.

(Ch. 1) A hollow cube (see figure on the right) has six square (正方形) sides. There are no charges inside. Five sides are grounded. The sixth side, insulated from the others, is held at a constant potential ϕ_0. Find the potential at the center of the cube by using Eq. (1.44).
[Hint. There is no need to calculate the Green function G($\boldsymbol{x}$, $\boldsymbol{x}'$).]



(Ch. 1). (a) Find the charge distribution ρ that would produce the electrostatic potential

$$\phi(r) = \frac{q}{4\pi\varepsilon_0 r} e^{-\frac{r}{a}}$$

(b) Calculate the total charge.

(Ch. 1) The time-averaged potential of a neutral hydrogen atom is given by

$$\phi(r) = \frac{q}{4\pi\varepsilon_0}\frac{e^{-\alpha r}}{r}\left(1+\frac{\alpha r}{2}\right)$$

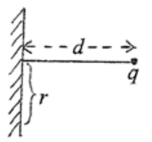
where q is the magnitude of the electron charge, and $\alpha^{-1} = a_0/2$, a_0 being the Bohr radius. Find the distribution of charge (both continuous and discrete) that will give this potential and interpret your result physically.

(Ch. 1) Let $G(\mathbf{x}, \mathbf{x}')$ be the Green function for the Laplacian operator ∇^2, subject to the boundary condition $G(\mathbf{x}, \mathbf{x}') = 0$ on an arbitrary boundary surface. Use Green's theorem to prove that $G(\mathbf{x}, \mathbf{x}')$ satisfies the symmetry relation: $G(\mathbf{x}, \mathbf{x}') = G(\mathbf{x}', \mathbf{x})$.

(Ch. 1) Derive Coulomb's law, $\phi(\mathbf{x}) = \frac{1}{4\pi\varepsilon_0}\int_V \frac{\rho(\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|} d^3x'$, from
$\nabla\cdot\mathbf{E} = \frac{\rho}{\varepsilon_0}$ and $\nabla\times\mathbf{E} = 0$ by applying Green's second identity.
$\mathbf{E} = -\nabla\phi$ and the region of interest is infinite space.

(Ch. 2) A point charge q is brought from infinity to a distance d from an infinite, grounded, plane conductor. If the induced surface charge on the conductor is now "frozen" in place and the point charge is removed, what is the energy in the remaining electric field?

(Ch. 2) A point charge q is situated a distance d from an infinite, flat, and grounded conducting surface (see figure below). Find the charge density σ on the conducting surface as a function of charge q, and distances d and r, where d and r are indicated in the figure.



(Ch. 2) Consider an infinite set of complex functions $U_n(\xi)$ $(n = 1, 2, \cdots)$ in the interval $a \le \xi \le b$. They are orthogonal in the sense

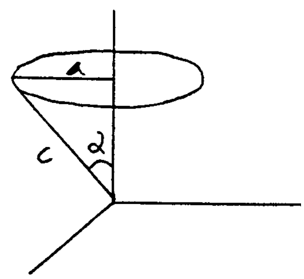
$$\int_a^b U_n(\xi)U_m^*(\xi)d\xi = \begin{cases} = 0, & \text{if } m \ne n \\ \ne 0, & \text{if } m = n \end{cases}$$

Prove these functions are linearly independent.

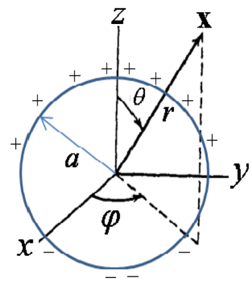
(Ch. 3) A total charge q is uniformly distributed along a thin circular ring of radius a as shown in Jackson Fig. 3.4 on p. 103 (or the figure to the right).

(a) Write down the volume charge density (ρ) in spherical coordinates.

(b) Instead of using the method on p.103, use Jackson Eq. (3.70) to derive the electrostatic potential of the charged ring.



(Ch. 3) A surface charge density $\sigma(\theta) = \sigma_0 \cos\theta$ is glued to the surface of a spherical shell of radius R (σ_0 is a constant and θ is the polar angle in the spherical coordinate system.) There is a vacuum, with no charges, both inside and outside of the shell. Calculate the electrostatic potential inside and outside of the spherical shell.



(Chaps. 1, 3, 4) A spherical surface of radius a has a surface charge density

$$\sigma(\mathbf{x}) = 3\varepsilon_0 E_0 \cos\theta \text{ (in spherical coordinates),}$$

which corresponds to a volume charge density: $\rho(\mathbf{x}) = 3\varepsilon_0 E_0 \cos\theta\,\delta(r-a)$.

(a) Calculate the dipole moment by using $\mathbf{p} = \int \mathbf{x}\rho(\mathbf{x})d^3x$ [(4.7)]

Hint : Express $\mathbf{x}$ in spherical coordinates.

(b) Using (1.17) and (3.70), calculate the electrostatic potential ϕ due to σ at $r < a$ and $r > a$.

(Ch. 3) A conducting sphere of radius a and with net charge Q is placed in a uniform electric field $E_0\boldsymbol{e}_z$. Use the method of expansion, find the potential at all points outside the sphere and the surface charge distribution on the sphere.

(Ch.3) An operator L is Hermitian if $\int u_1^*(\mathbf{x})Lu_2(\mathbf{x})d^3x = [\int u_2^*(\mathbf{x})Lu_1(\mathbf{x})d^3x]^*$, where u_1 and u_2 are arbitrary functions satisfying homogeneous boundary conditions. Prove that the Green function $G(\mathbf{x}, \mathbf{x}')$ for a Hermitian differential operator satisfies the symmetry relation:

$$G(\mathbf{x}, \mathbf{x}') = G^*(\mathbf{x}', \mathbf{x}).$$

(Ch. 3) Referring to Fig. 3.7 of Jackson, prove the following relation:

$$\cos\gamma = \cos\theta\cos\theta' + \sin\theta\sin\theta'\cos(\phi - \phi').$$

(Ch. 3) (a) An infinitely long rectangular waveguide extends in the z direction. The conducting walls are defined by the planes $x = 0$, $x = a$, $y = 0$, $y = b$ (see figure). Show that inside the waveguide the Green's function suitable for Dirichlet boundary conditions is given by

$$G(\mathbf{x}, \mathbf{x}') = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} \frac{8\pi}{ab\gamma_{nm}} e^{-\gamma_{nm}(z_> - z_<)} \sin(\frac{n\pi}{a}x)\sin(\frac{n\pi}{a}x')\sin(\frac{m\pi}{b}y)\sin(\frac{m\pi}{b}y')$$

where $\gamma_{nm}^2 = (\frac{n\pi}{a})^2 + (\frac{m\pi}{b})^2$.

(b) A charge q is placed at the center of the waveguide at $(x, y, z) = (a/2, b/2, 0)$, and the walls are grounded. What is the potential inside the waveguide? What is the asymptotic form of the potential along the center of the waveguide $(a/2, b/2, z)$ as $z \rightarrow \infty$?

(c) If the charge is removed and the wall at $x = 0$ is held at potential V with the other walls grounded, what is the potential in the space inside the waveguide?

(Ch. 4) (a) Refer to the mechanism of dipole formation discussed in Sec. 4.6. Find the energy required to induce a dipole on an atomic or molecular charge e by an electric field $\mathbf{E}$.

(b) From $W = \frac{1}{2}\int \mathbf{E}\cdot\mathbf{D}d^3x$ [(4.89)], we deduce that, in a dielectric, the energy density due to the presence of $\mathbf{E}$ is $w = \frac{1}{2}\mathbf{E}\cdot\mathbf{D}$. Derive this relation using the result in Part (a).

(Ch. 4) Show that, inside a *linear* and *uniform* dielectric medium of permittivity ε, the polarization charge density (ρ_p) is related to the net charge density (ρ_{net}) by

$$\rho_p = \frac{\varepsilon_0 - \varepsilon}{\varepsilon}\rho_{net},$$

where ε_0 is the permittivity of free space.

(Ch. 4) What is the relative electrostatic field energy between two dipoles $\mathbf{p}_1$ and $\mathbf{p}_2$ located, respectively, at $\mathbf{x}_1$ and $\mathbf{x}_2$?

(Ch. 4) An ideal dipole $\mathbf{p} = (3\mathbf{e}_x + 4\mathbf{e}_y)$ Coulomb-meter is located at $\mathbf{x} = 0$ in free space. Find the vector force it exerts on an electron located at $\mathbf{x} = (4\mathbf{e}_x + 3\mathbf{e}_y)$ meters. [e = 1.6x10^{-19} Coulomb. The permittivity of free space (ε_0) is given in Jackson, p. 782.]

(Ch. 4) A charge distribution is given by $\rho(\boldsymbol{x}) = -\boldsymbol{p}_0 \cdot \nabla\delta(\boldsymbol{x})$, where $\boldsymbol{p}_0$ is a constant vector and $\delta(\boldsymbol{x})$ is the Dirac delta function.

(a) Calculate the electric monopole moment.

(b) Calculate the electric dipole moment.

(Ch. 4) A cylindrical rod of radius a and infinite length possesses an electric polarization $\boldsymbol{P}$ given by $\boldsymbol{P}(r) = p_0 r\boldsymbol{e}_r$, where p_0 is a constant, r is the radial distance from the axis of the rod, and $\boldsymbol{e}_r$ is a unit vector along r. Calculate the volume polarization charge density ρ_p inside the rod and the surface polarization charge density σ_p on the rod. (Hint: $\rho_p = -\nabla\cdot\mathbf{P}$)

(Ch. 4) Consider a dielectric sphere of radius a with polarization $\boldsymbol{P} = Kr\boldsymbol{e}_r$, where K is a constant, r is the distance from the center, and $\boldsymbol{e}_r$ is a unit vector in the radial direction.

(a) Find the polarization charge density.

(b) Show by direct integration that the total polarization charge is 0.

(c) What is the electric field inside the sphere?

(Ch. 4) Show that, in the absence of free charges, the polarization charge density ρ_p in an inhomogeneous, isotropic dielectric [$\varepsilon = \varepsilon(\mathbf{x})$] is given by

$$\rho_p = -\frac{\varepsilon_0}{\varepsilon}\boldsymbol{E}\cdot\nabla\varepsilon$$

(Ch. 1-4) A dielectric medium could have any or none of the following properties:

(A) linear; (B) isotropic; and (C) uniform.

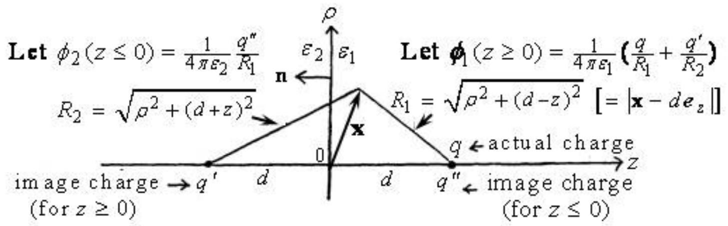
Of the properties listed above, what are required to derive the following electrostatic relations: (12%, 3 points will be deducted for each wrong or missing answer.)

(a) $\mathbf{E} = -\nabla\phi$

(b) $\nabla^2\phi = -\frac{\rho}{\varepsilon}$

(c) $W = \frac{1}{2}\int \mathbf{E}\cdot\mathbf{D}d^3x$

(Ch. 4) Refer to the problem on Jackson, p. 155. Two semi-infinite dielectrics have an interface plane at $z = 0$. A point charge q is at $z = d$. Use the results in Jackson [Eqs. (4.43)-(4.45), find ρ_{pol} in the region $z>0$ and $z < 0$ [no need to find σ_{pol} at $z = 0$, which is given by (4.47)].



(Chap. 4) A point charge q is located in an infinite, uniform medium of permittivity ε.
Put the charge at the origin of coordinates. Calculate

(a) The potential $\phi(\mathbf{x})$ everywhere.

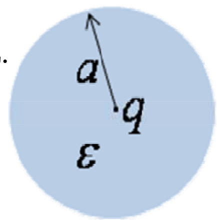
(b) The polarization charge density $\rho_{pol}(\mathbf{x})$ [$= -\nabla\cdot\mathbf{P}(\mathbf{x})$].

(Ch. 4) A point charge q is placed at the center of a uniform dielectric sphere of radius a and permitivity ε. The outside is free space (permitivity ε_0). By symmetry, $\mathbf{E} = E_r(r)\mathbf{e}_r$.

(a) Derive $E_r(r)$ everywhere [i.e. $E_r^{out}(r)$ and $E_r^{in}(r)$].

(b) The polarization charge density is given by $\rho_{pol} = -\nabla\cdot\mathbf{P}$, where $\mathbf{P} = (\varepsilon - \varepsilon_0)\mathbf{E}$. Derive the surface polarization charge density (σ_{pol}) at $r = a$.

(c) Find other polarization charge(s), if any. If there is none, write 0.



(Ch. 5) A charged particle of charge q and mass m is moving in a straight line at constant velocity $\mathbf{v}_0$ under the influence of a uniform static electric field $E_0\mathbf{e}_y$ and uniform static magnetic field $B_0\mathbf{e}_z$.

(a) Find $\mathbf{v}_0$ (magnitude and direction) from the nonrelativistic equation of motion

$$\frac{d}{dt}\mathbf{v} = \frac{q}{m}(E_0\mathbf{e}_y + \mathbf{v}\times B_0\mathbf{e}_z)$$

(b) What is the condition of validity of your solution in (a)?

(c) While the charged particle is moving in a straight line at the constant velocity $\mathbf{v}_0$ determined in (a), a perturbed velocity $v_1\mathbf{e}_x$ is suddenly added to the particle at $t = 0$. Find the subsequent velocity $\mathbf{v}(t)$ of the particle as a function of time. [Hint: Use the equation of motion in (a) and write $\mathbf{v}(t) = \mathbf{v}_0 + \delta\mathbf{v}(t)$]

(d) Is the validity of your result in (c) subject to the condition $v_1 << v_0$? If your answer is yes, state the reason.

(Ch. 5) Explain why the derivation of the magnetic field energy in Jackson Sec. 5.16 is valid only when the current **J** is established at an infinitesimally small rate.

(Ch. 5) (a) Can the magnetization current $\boldsymbol{J}_m$ in Jackson Eq. (5.79) be formed of moving charges? (No explanation is required. Just answer "yes" or "no")

(b) Prove mathematically that $\boldsymbol{J}_m$ can never produce a charge density.

(Ch. 5) The vector potential arising from a uniform distribution of DC current in an infinite cylinder of radius a can be written

$$\boldsymbol{A} = A(r)\,\boldsymbol{e}_z,$$

where $\boldsymbol{e}_z$ is along the axis of the cylinder. Assume that the total current is I, calculate $A(r)$.

(Ch. 5) The two rails of a railroad track, insulated from each other and from the ground, are connected by a voltmeter. What is the reading on the voltmeter when a train is approaching at the speed of 180 km/hr, assuming that the vertical component of the earth's magnetic field is 0.2 gauss and that the tracks are separated by one meter?

(Ch. 5) Consider a thin circular loop (of radius a) carrying a steady current I. Let its axis be the z-axis (see Jackson Fig. 5.5).

(a) What is the magnetic induction **B** on the z-axis?

(b) Evaluate the integral

$$\int_{-\infty}^{\infty} B_z(z)dz,$$

where $B_z(z)$ is the z-component of **B**.

(c) If a = 1 cm and I = 1 Amp, what is the value of B_z at the center of the loop? (The permeability of vacuum is given in Jackson, p. 782.)

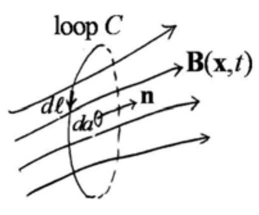
(Ch. 5) Assuming that the magnetic induction $\mathbf{B}(\mathbf{x}, t)$ is known, does the Faraday's law constitute a sufficient condition for specifying the electric field $\boldsymbol{E}(\mathbf{x}, t)$? Why?

(Ch. 5) Two identical, perfectly conducting loops are far apart and share the same axis. Each has self-inductance L and a current I flowing in the same direction, so the energy in each loop is $\frac{1}{2}\mathrm{LI}^2$. They are then brought together and superposed.

(a) What is the final current in each loop?

(b) What is work done in bringing the two loops together? Is the work done *on* the loops or *by* the loops?

(Ch. 5) Answer the following questions about the Faraday's law: $\oint_C \mathbf{E}\cdot d\ell = -\int_S \frac{\partial \mathbf{B}}{\partial t}\cdot \mathbf{n}da$. No explanation is required.



(a) If loop C is a closed electrical circuit formed of a perfectly conducting wire, what is the value of $\oint_C \mathbf{E}\cdot d\ell$?

(b) Assume the loop in (a) is an open circuit with a narrow gap in the perfectly conducting wire and $\int_S \frac{\partial \mathbf{B}}{\partial t}\cdot \mathbf{n}da \neq 0$. Where is the **E**-filed?

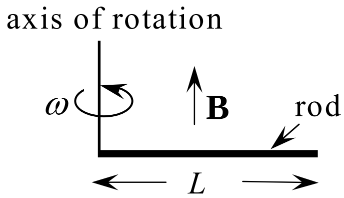
(c) Consider the case in (b) again. What is the magnitude of ΔV ($=\int \mathbf{E}\cdot d\ell$) across the gap? What is the ΔV along the total length of the perfectly conducting wire?

(Ch. 5) Prove that the magnetic dipole moment (**m**) for a localized static current $\mathbf{J}(\mathbf{x})$ is independent of the point of reference.

(Ch. 5) Assume that the earth magnetic field is caused by a small current loop located at the center of the earth. Given that the field near the north and south poles is 0.8 Gauss (oriented perpendicular to the ground) and that the earth radius is $R = 6\times10^6$ m, calculate the strength of the magnetic dipole moment of the small current loop. You may use any equation in Jackson without derivation.

(Ch. 5) A particle of mass m and magnetic dipole moment $\mathbf{m}_1$ moves in a circular orbit of radius R about a stationary magnetic dipole of moment $\mathbf{m}_2$ located at the center of the circle. The physical sizes of both diploes are much smaller than R. The vectors $\mathbf{m}_1$ and $\mathbf{m}_2$ are antiparallel to each other and perpendicular to the plane of the circle. Find the velocity v of the orbiting dipole.

(Ch. 5) As shown in the figure, a copper rod of length L is pivoted at one end and rotates with constant angular velocity ω about an axis perpendicular to its length . A uniform magnetic induction **B** is applied parallel to the axis of rotation. Calculate the potential difference between the ends of the rod?



(Ch. 1-5) Answer "yes" or "no" to the following statements (no explanation is required). Notations follow Jackson. Two points for each correct answer; no penalty for wrong answers.

(1) Eq. (1.13) gives exactly the same information as Eq. (1.5).

(2) $\varepsilon_0 E^2(\mathbf{x})$ and $q\delta(\mathbf{x}-\mathbf{x}_0)\Phi(\mathbf{x})$ have the same dimension.

(3) Legendre function of the first kind $P_\nu(x)$ diverges at $x = 1$ unless ν is an integer.

(4) Bessel function of the first kind $J_n(x)$ is an oscillatory function of x (x is real and n is a positive integer).

(5) Electric polarization and electric dipole moment do not have the same dimension.

(6) The polarization charge density ($\rho_{pol} = -\nabla\cdot\mathbf{P}$) is only a convenient definition. It does not represent real charges.

(7) $\Phi(\mathbf{x})$ in (4.84) does not include contribution from the polarization charge.

(8) Eq. (4.89) can be applied to a linear, anisotropic, and nonuniform dielectric medium.

(9) In magnetostatics, $\int_v \mathbf{J} d^3x = 0$ for an arbitrary volume (v) of integration.

(10) Eq. (5.22) gives exactly the same information as Eq. (5.24).

(11) Eq. (5.147) is derived from Eq. (5.146) under the assumption that the volume of integration is infinity.

(12) A static magnetic field can penetrate through a conductor with $\mu = \mu_0$ and finite conductivity as if there were no conductor present.

(Ch. 5, 6) Assume that, at $t = 0$, a solid conducting sphere of uniform conductivity σ and permittivity ε has a uniform net charge density ρ_0. There is no external field of any kind.

(a) Show that, at $t > 0$, the net charge density (ρ_{net}) and the electric field ($\mathbf{E}$) are given by

$$\rho_{net} = \rho_0 e^{-\frac{\sigma t}{\varepsilon}} \text{ and } \mathbf{E} = \frac{\rho_0 r}{3\varepsilon} e^{-\frac{\sigma t}{\varepsilon}} \mathbf{e}_r$$

Hint: Use $\nabla \cdot \mathbf{D} = \rho_{net}$, $\mathbf{J}_{net} = \sigma \mathbf{E}$, and $\nabla \cdot \mathbf{J}_{net} + \frac{\partial \rho_{net}}{\partial t} = 0$

(b) Show that, at every point in the conductor, the dissipated Ohmic power is equal to the rate of reduction of the electric field energy at that point.

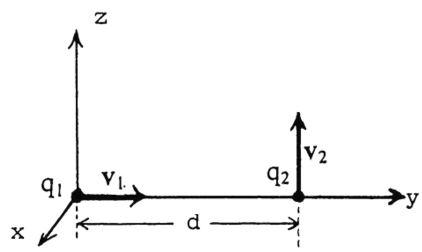
(c) Show that no magnetic field will be generated anywhere at any time.

(d) If the radius of the sphere is R, show that the total charge that eventually accumulates on the surface is $Q = \frac{4\pi}{3} R^3 \rho_o$

(Ch. 5, 6) Consider two point charges q_1 and q_2 with velocities $\mathbf{v}_1 = v_1 \mathbf{e}_y$ and $\mathbf{v}_2 = v_2 \mathbf{e}_z$, respectively (see figure). Both charges are on the y-axis and separated by a distance d. If v_1 and $v_2 \ll$ c (as is assumed here), the electric and magnetic fields due to q_1 and q_2 can be approximated by the static laws [e.g. Eqs. (5.14) and (5.15) in Jackson].

(a) Calculate the force on q_1 due to q_2 and the force on q_2 due to q_1.

(b) Are the forces on the two charges equal in magnitude and opposite in direction as required by Newton's third law? Explain your answer in terms of the conservation of linear momentum.



(Ch. 6) A conducting surface has a static surface charge density σ. For simplicity, assume that the surface is flat. Use the Maxwell stress tensor [Jackson Eqs. (6.120) and (6.122)] to show that the force per unit area on the conducting surface is $\sigma^2/(2\varepsilon_0)$.

(Ch. 6) Verify Poynting's theorem [Jackson Eq. (6.107)] for the case of a long, straight conducting wire of radius a and conductivity σ, which carries a direct current I. [The resistance per unit length of the wire is $R = 1/(\sigma\pi a^2)$]

(Ch. 6) In order to evaluate the electromagnetic fields at position $\boldsymbol{x}$ and time t produced by a charged particle whose orbit as a function of time is given by $\boldsymbol{r}(t)$, we need to know the retarded time t'. Write down the equation for the determination of t'. Derivation is not required.

(Ch. 6) Consider a possible solution to Maxwell's equations (with $\rho = \boldsymbol{J} = 0$, $\varepsilon = \varepsilon_o$, and $\mu = \mu_o$) given by

$$\boldsymbol{A}(\boldsymbol{x},t) = \boldsymbol{A}_0 e^{-i\omega t + i\boldsymbol{k}\cdot\boldsymbol{x}}, \phi(\boldsymbol{x},t) = 0,$$

where $\boldsymbol{A}$ is the vector potential and ϕ is the scalar potential. Give the constraint(s) on $\boldsymbol{A}_0$, $\boldsymbol{k}$ and ω as imposed by each of the Maxwell's equations:

(a) $\nabla \cdot \mathbf{B} = 0$, (b) $\nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = 0$, (c) $\nabla \cdot \mathbf{E} = 0$, and (d) $\nabla \times \mathbf{B} + \varepsilon_0 \mu_0 \frac{\partial \mathbf{E}}{\partial t} = 0$.

(Ch. 6) A spaceship with a mass of 1.5×10^3 kg (including an astronaut) is stationary in outer space with negligible gravitational force acting on it. The astronaut then turns on a 10 kW laser beam pointing to a fixed direction. Use Jackson Eqs. (6.118) and (6.122) to calculate the speed of the space ship after the laser has been on for one day.

(Ch. 6) (a) Write down the four Maxwell equations.

(b) Show that conservation of charge is implicit in this set of equations.

(c) Show that the electric field **E** and magnetic induction **B** can be represented by a scalar potential φ and a vector potential **A**. Specific expressions for φ and **A** are not required.

(Ch. 6) (a) Write down the four Maxwell's equations in the absence of dielectric or magnetic materials. State your system of units. In all of the following you must prove your answer.

(b) If the signs of all the source charges are reversed, what happens to the electric and magnetic fields $\boldsymbol{E}$ and $\boldsymbol{B}$?

(c) If the system is space inverted, i.e., $\boldsymbol{x} \rightarrow \boldsymbol{x}' = -\boldsymbol{x}$, what happens to the charge and current densities, ρ and $\boldsymbol{j}$, and to $\boldsymbol{E}$ and $\boldsymbol{B}$?

(d) If the system is time reversed, i.e., $t \rightarrow t' = -t$, what happens to ρ, $\boldsymbol{j}$, $\boldsymbol{E}$ and $\boldsymbol{B}$?

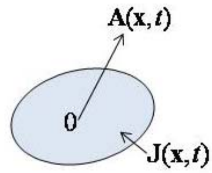
(Ch. 6) (a) Assume $\mathbf{J}(t) = \mathrm{Re}[\mathbf{J}_0 e^{-i\omega t}]$ and $\mathbf{E}(t) = \mathrm{Re}[\mathbf{E}_0 e^{-i\omega t}]$, where $\mathbf{J}_0$ and $\mathbf{E}_0$ are complex constants and ω is a real constant. Show that the time-averaged value of $\mathbf{J}(t)\cdot\mathbf{E}(t)$ is given by $\frac{1}{2}\mathrm{Re}[\mathbf{J}_0\cdot\mathbf{E}_0^*]$.

(b) Assume $\varepsilon = \varepsilon' + i\varepsilon''$ is a complex number. Show $\mathrm{Re}\sqrt{\varepsilon}\cdot\mathrm{Im}\sqrt{\varepsilon} = \frac{1}{2}\varepsilon''$

(Ch. 6) A localized source $\mathbf{J}(\mathbf{x},t) = \mathbf{J}(\mathbf{x})\cos\omega t$ produces a vector potential $\mathbf{A}(\mathbf{x},t)$.

(a) Use Jackson Eq. (6.48) to express $\mathbf{A}(\mathbf{x},t)$ as an integral representing the superposition of an infinite number of components originating from all points of the source.

(b) Explain how the effect of "time retardation" is reflected in the phase angle of each component in $\mathbf{A}(\mathbf{x},t)$.



(Ch. 6, 7) A plane electromagnetic wave incident normally from the free space onto a flat surface is partly reflected and partly absorbed. Use Jackson Eqs. (6-122) and (6-120) to prove that the radiation pressure on the surface is equal to the field energy density outside the surface no matter what fraction of the incident power is reflected.

(Ch. 6, 7) (a) A plane electromagnetic wave is incident normally from the free space onto the flat surface of a wave-absorbing material, which absorbs all the incident power. Use Jackson Eqs. (6-122) and (6-120) to calculate the instantaneous radiation pressure exerted on the material in terms of the instantaneous wave fields E and B on the surface.

(b) What is the difference (if any) between the time-averaged radiation pressures exerted by a *linearly* polarized plane wave and a *circularly* polarized plane wave with the same peak fields E_0 and B_0?

(Ch. 6, 7) A plane electromagnetic wave with instantaneous fields $\mathbf{E}_i$ and $\mathbf{B}_i$ is incident normally from the free space onto a perfectly conducting flat surface. The reflected wave has instantaneous fields $\mathbf{E}_r$ and $\mathbf{B}_r$. The surface is stationary.

(a) What is the relation between $\mathbf{E}_r$ and $\mathbf{E}_i$ on the surface? Give your reason.

(b) What is the relation between $\mathbf{B}_r$ and $\mathbf{B}_i$ on the surface? Give your reason.

(c) Use Jackson Eqs. (6-122) and (6-120) to calculate the instantaneous radiation pressure exerted on the conductor in terms of the incident fields.

(Ch. 6, 7) Answer questions (a)-(d) below regarding the assumptions made for the derivation of Jackson Eq. (6.107); namely, the macroscopic medium is <u>linear in its electrical property</u> and it has <u>negligible dispersion</u> and <u>negligible loss</u>.

(a) Write down the equation which expresses the linear electrical property of the medium in the ω space.

(b) Using your answer in (a) and Jackson Eq. (7.51), show that the linear property can be expressed in time-space as $\mathbf{D}(t) = \varepsilon\mathbf{E}(t)$ under the assumptions of negligible dispersion and negligible loss, i.e. when $\varepsilon(\omega) = \varepsilon(\omega_0)$ and $\gamma_j = 0$, where ω_0 is a constant.

(c) Why is the assumption of "negligible loss" required for the derivation in (b)?

(d) Assume an electromagnetic signal is propagating in the medium. What is the condition on the signal in order for the dispersive property of the medium to be negligible? Show the condition in an ω-space plot.

(Ch. 6,7) Answer "yes" or "no" to the following statements (no explanation is required).
(18%, 3 points will be deducted for each wrong answer.)

(a) Poynting's theorem applies to static as well as time-dependent fields.

(b) All 4 of the Maxwell equations have been used in the derivation of Jackson Eqs. (6.15) and (6.16).

(c) At any given point in space, the Poynting vector of a linearly polarized plane wave varies with time.

(d) The retarded Green function $G^{(+)}$ in Jackson Eq. (6.44) is applicable to both moving charges and antennas.

(e) The retarded Green function $G^{(+)}$ in Jackson Eq. (6.44) is not applicable to a dispersive medium.

(f) The dispersive property of a medium as exhibited in Jackson Eq. (7.51) is due to an inertial effect.

(Ch. 7) (a) Starting from the Maxwell equations [Jackson Eq. (6.6)], derive the dispersion relation (i.e. the relation between the wave frequency ω and the propagation constant k) for a (homogeneous) plane electromagnetic wave of frequency ω in an infinite and uniform medium of conductivity σ, electric permittivity ε, and magnetic permeability μ.

[vector formula: $\nabla\times(\nabla\times\mathbf{A}) = \nabla(\nabla\bullet\mathbf{A}) - \nabla^2\mathbf{A}$]

(b) From the result in (a), derive the expression for the skin depth δ of a good conductor.

(Ch. 7) Consider a plane wave propagating in a dielectric medium characterized by a complex electric permittivity of $\varepsilon = \varepsilon_0(1 + 0.001i)$ and a complex magnetic permeability of $\mu = \mu_0(1 + 0.001i)$. Calculate the fractional power loss over a distance of one wavelength.

(Ch. 7) Using Jackson Eqs. (7.4), (7.56), and (7.58) and assuming $\omega \ll \gamma_0$, derive the skin depth (δ) of a good conductor.

(Ch. 7) Show that the instantaneous Poynting vector of a circularly polarized plane electromagnetic wave of constant amplitude is independent of time.

(Ch. 7) A plane electromagnetic wave with vector fields $\mathbf{E}_i$ and $\mathbf{B}_i$ is incident normally from free space upon a perfectly conducting flat surface. The reflected wave has vector fields $\mathbf{E}_r$ and $\mathbf{B}_r$.

(d) What is the relation between $\mathbf{E}_r$ and $\mathbf{E}_i$ on the surface? Give your reason.
(e) What is the relation between $\mathbf{B}_r$ and $\mathbf{B}_i$ on the surface? Give your reason.

(Ch. 7) A highly relativistic electron is moving in the direction of propagation of a plane electromagnetic wave in free space. Estimate the ratio of the maximum electric force to the maximum magnetic force on the electron.

(Ch. 7) (a) Calculate the peak electric field (E_0) associated with a linearly polarized laser beam (in free space) having a time-averaged energy density of 10^6 joules/cm^3.
(b) What is the peak magnetic induction (B_0) of this laser beam?

Note: Values of electrical permittivity (ε_0) and magnetic permeability (μ_0) for the vacuum can be found in Jackson, p. 782.

(Ch. 7) (a) The electron has a charge of $e = 1.6\times10^{-19}$ C. Assume it has a spherical shape with a radius of 3×10^{-15} m and its charge is uniformly distributed in its volume. Calculate the electric field on the surface of the electron.
(b) If the result in (a) is the peak electric field of a monochromatic plane electromagnetic wave in free space. What is the peak magnetic induction (B) of this wave? What is the peak power per unit area of this wave?

Useful information:
$\oint_S \mathbf{E}\cdot\mathbf{n}\, da = \frac{1}{\varepsilon_0}\int_V \rho(\mathbf{x})d^3x$ [Gauss law]; $\nabla\times\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t}$ [Faraday's law];
$\mathbf{S} = \mathbf{E}\times\mathbf{H}$ [Poynting vector]; $\varepsilon_0 = 8.85\times10^{-12}$ F/m; $\mu_0 = 1.26\times10^{-6}$ H/m.

(Ch. 7) As implied in Jackson Eqs. (6.15) and (6.16), the electromagnetic radiation is generated by electrical charges and currents. In the equations of Jackson Sec. 7.3, there are no explicit expressions for charges and currents; however, the results show that a reflected wave is generated when a plane wave is incident from one dielectric medium into another dielectric medium. Consider a wave incident from a vacuum into a dielectric medium with $\mu = \mu_0$ and $\varepsilon \neq \varepsilon_0$ (a special case of Sec. 7.3). Explain qualitatively from the perspective of Eqs. (6.15) and (6.16) how the reflected wave is generated?

(Ch. 7) (a) A charge-neutral, uniform plasma has N electrons per unit volume. Assume that the ions remain stationary in the presence of an ac electric field which varies with time as exp(-iωt). Using the equation of motion for the electrons and neglecting collisions, show that the conductivity (σ) of the medium is $\sigma = iNe^2/\omega m$, where e is the magnitude of the electron charge and m is the electron mass.

(b) Using the Maxwell equations, derive the dispersion relation for a plane electromagnetic wave in this medium. What is the "plasma frequency"?

(c) It is known that radio waves can be totally reflected from the ionosphere. Under what condition will this happen?

(Ch. 7) (a) Derive the AC conductivity (σ) of a conductor containing N free electrons per unit volume which collide at an average frequency of γ. Start from the equation of motion of free electrons and neglect the effect of bound electrons.

(b) If the electric field is given by $E = E_0\cos(\omega t+\alpha)$, where E_0, ω, and α are real constants, what are the amplitude and phase (relative to that of E) of the current density?

(Ch. 7) Inside the good conductor, the electromagnetic wave damps by a factor of $1/e$ over a distance of the skin depth δ, where $\delta = \sqrt{\frac{2}{\mu\sigma\omega}}$.

(a) Calculate the skin depth of sea water ($\sigma \approx 10$ /Ω-m, $\mu = 1.26\times10^{-6}$ H/m) at the frequency of 10^{10} Hz.

(b) The sunlight has a frequency $\gg 10^{10}$ Hz. Why can it penetrate deep into sea water?

(Ch. 7)(a) A one-dimensional wave packet is propagating in a uniform medium characterized by a dispersion relation $\omega = \omega(k)$, where ω is the frequency and k is the wave number. Under what condition will the wave packet remain undistorted in shape? Express the condition analytically.

(b) What is your physical interpretation of the result in (a)?

(Ch. 7) Consider a medium characterized by the *generalized* electric permittivity ε in Jackson Eqs. (7.51) and (7.56). Answer the following questions briefly and to the point.

(a) In the derivation of ε, what are the approximations made on the electron binding force and on the electric field, which result in the *linear* property of the medium?

(b) What makes the medium dispersive?

(c) What makes the medium lossy?

(d) In the DC limit ($\omega \to 0$), we have $\varepsilon \to \infty$ due to the free electrons. Hence, the expression for ε is no longer valid. What is the physical reason for the breakdown?

(e) Explain *physically* why a free electron is much more effective in damping the electromagnetic wave than a bound electron.

(Ch. 7) Consider Eq. (7.49) of Jackson: $m(\ddot{\mathbf{x}} + \gamma\dot{\mathbf{x}} + \omega_0^2\mathbf{x}) = -e\mathbf{E}(\mathbf{x}, t)$

(a) Solve it for $\mathbf{x}(t)$ by making the appropriate assumption on $\mathbf{E}(\mathbf{x}, t)$.

(b) You will find from your solution in (a) that, if $\mathbf{E}(\mathbf{x},t) = 0$, there will be a different solution for $\mathbf{x}(t)$ under a certain condition. What is this condition?

(c) Let $\mathbf{E}(\mathbf{x},t) = 0$ and assume $\omega_0 \gg \gamma$, find the new solution for $\mathbf{x}(t)$ including the effect of γ.

(d) Physically, what does the solution in (c) represent?

(Ch. 7) Let $\mathbf{E}_0$ be a complex vector and $\mathbf{k} = k\mathbf{n}$, where k is a complex number and $\mathbf{n}$ is a real vector. Prove that if $\mathbf{k} \times \mathbf{E}_0 = 0$, then $\mathbf{k} \times \mathbf{E}_0^* = 0$, where $\mathbf{E}_0^*$ is the complex conjugate of $\mathbf{E}_0$.

(Ch. 7) A measuring device is disturbed by the following influences. How would you separately protect the device from each of the following influences?

(a) High-frequency electromagnetic fields.

(b) D.C. and low-frequency magnetic fields.

(Ch. 7) (a) The impedance of a medium is defined as $Z \equiv \sqrt{\frac{\mu}{\varepsilon}}$ (p. 297). Write the reflection coefficient (denote it by Γ) for normal incidence, as given in Eq. (7.42), in terms of Z for a wave incident from vacuum ($Z = Z_0$) onto a medium of impedance Z'.

(b) The generalized ε in Eq. (7.56) can be written $\varepsilon = \varepsilon_b + i\frac{\sigma}{\omega}$. For a good conductor, $\varepsilon \approx i\frac{\sigma}{\omega}$. Assume μ and σ of the conductor are real numbers. Express the impedance for a good conductor (denote it by Z_s) in terms of σ, μ, and ω.

(c) Let $\mu_{\text{copper}} = 1.26\times10^{-6}$ H/m and $\sigma_{\text{copper}} = 5.9\times10^{7}$ /Ω-m, calculate the Z_s of copper at $f = 2.5\times10^{9}$ Hz (microwave) and $f = 2.5\times10^{13}$ Hz (infrared).

(d) For the reflection coefficient Γ obtained in (a), let $Z_0 = 377\ \Omega$ and $Z' = Z_s$ of copper, calculate $|\Gamma|$ (amplitude of Γ) at $f = 2.5\times10^{9}$ Hz and 2.5×10^{13} Hz.

(e) Show that the fraction of the incident power absorbed by copper at 2.5×10^{13} Hz is approximately 100 times greater than that at 2.5×10^{9} Hz.

(Ch. 7) Consider the reflection/refraction model (Fig. 7.5) treated in Sec. 7.3. Assuming normal incidence (i.e. incident angle $i = 0$) and $\mu = \mu'$, find the ratio of transmitted power to incident power in terms of the indices of refraction of the two media (n and n').

(Ch.7) (a) The polarization charge density is given by $\rho_{\text{pol}} = -\nabla\cdot\mathbf{P}$. Use this relation to show that the polarization current is given by $\mathbf{J}_{\text{pol}} = \frac{\partial}{\partial t}\mathbf{P}$.
(b) Consider a plane electromagnetic wave propagating in a dielectric medium with a complex permittivity $\varepsilon = \varepsilon' + i\varepsilon''$ and real permeability μ. At a given position, the electric field is $\mathbf{E} = \mathbf{E}_0 e^{-i\omega t}$ and the polarization is $\mathbf{P} = (\varepsilon - \varepsilon_0)\mathbf{E}$. Calculate the time-averaged power (per unit volume) deposited to the dielectric medium at this position.

(Ch. 7) Refer to Jackson Fig. 7.5, a plane wave is incident normally (i.e. $i = 0$) from the free space onto the flat surface of a dielectric with $\varepsilon' = 9\varepsilon_0$ and $\mu' = \mu_0$.

(a) What fraction of the incident power is transmitted into the dielectric?

(b) What is the phase change of the reflected wave at the interface?

(Ch. 7) (a) Under what condition can we write the dispersion relation of a uniform plasma [Eq. (7.61)] approximately in the form of Eq. (7.95)?

(b) A wave packet of width L at $t = 0$ is propagating in a uniform plasma of plasma frequency ω_p. Assuming the condition in (a) and using the model of Sec. 7.9, find the width $L(t)$ of the wave packet at $t > 0$.

(Ch. 7) An X-ray is incident from a vacuum on the flat surface of a metal with a permeability of $\mu = \mu_0$. If the angle of incidence (with respect to the normal to the surface) is greater than a critical angle θ_0, the X-ray will be totally reflected. Calculate θ_0 by using the permittivity of metal given by Jackson Eq. (7.56), assuming that the angular frequency ω of the X-ray is greater than the plasma frequency ω_p of the conduction electrons, and neglecting the effects of collisions and bound electrons.